Iowa State University

Hydrodynamic effects for Optical Trapping

Mechanical Engineering Department

2023 JOINT MIDWEST & GREAT LAKES REGIONAL MEETING

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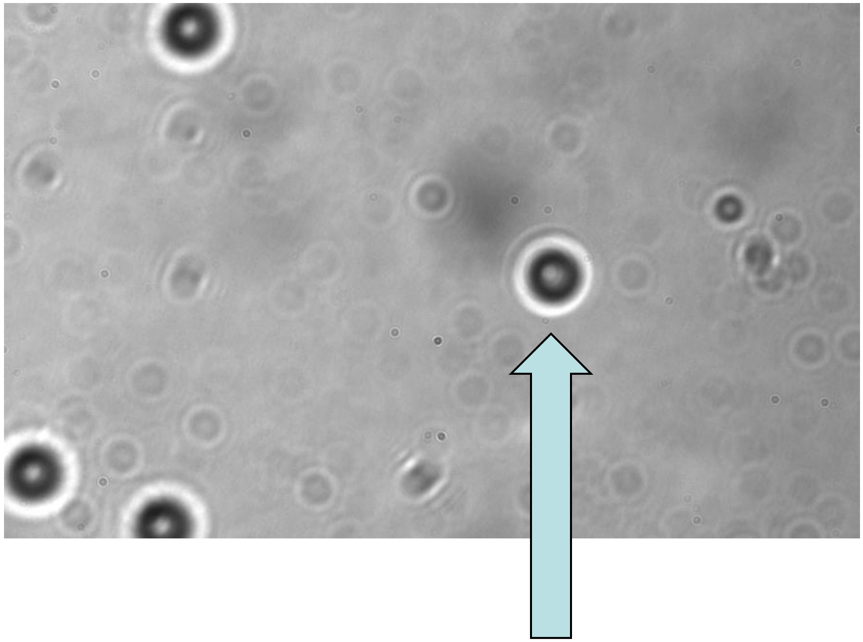
Motivation

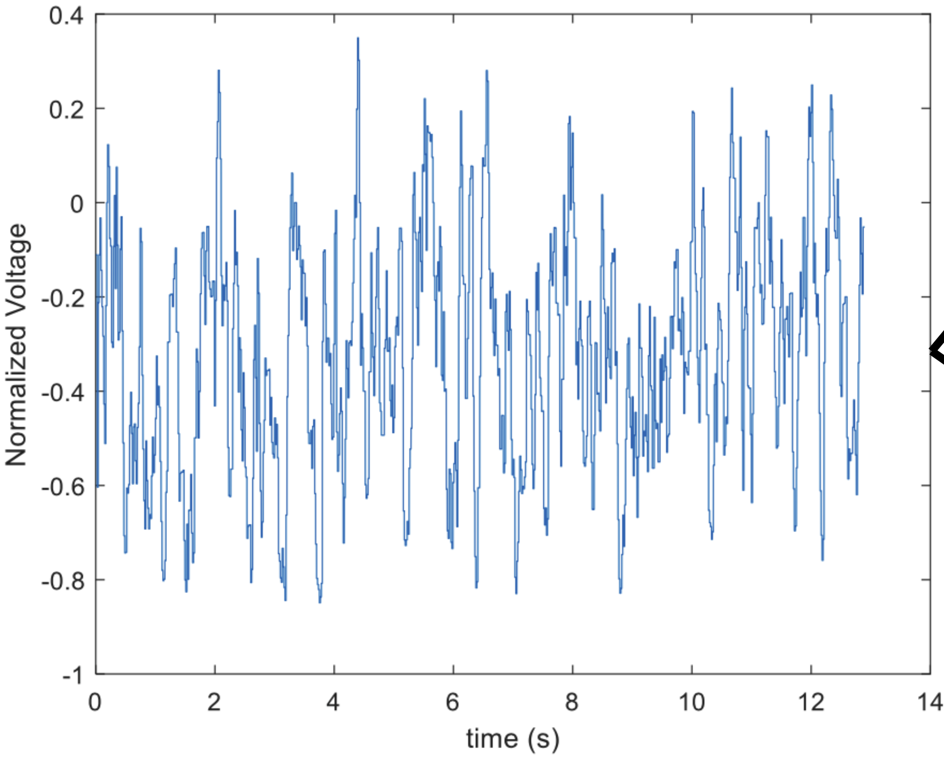
- Optical tweezers are used to characterize the rheological properties of fluids and gels containing trapped probe particle at a **wide range of corner frequency**.
- An accurate measurement of the medium viscosity is required for fundamental understanding of the dynamics of optically trapped particles.
- Hydrodynamic interaction with nearby wall is **complex** & is significant at **small time scale** influencing the viscosity.

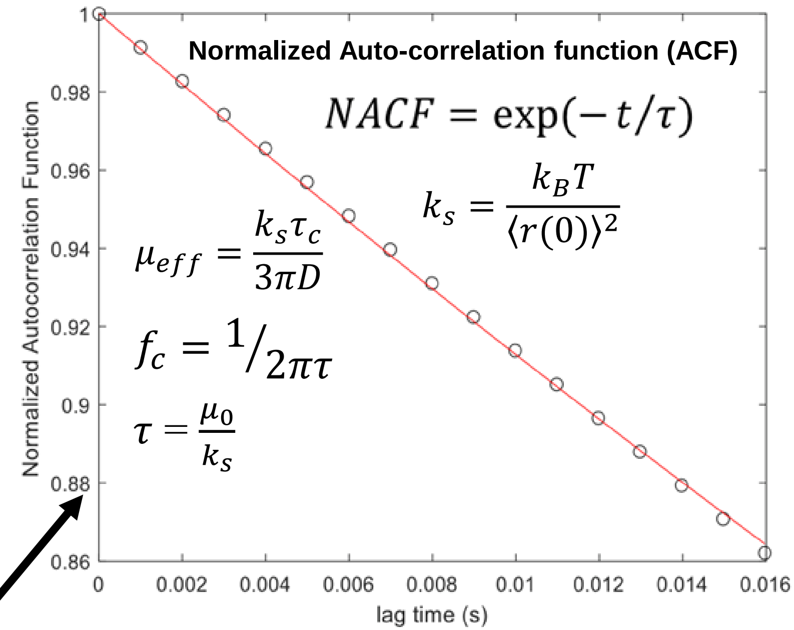
Experiments performed at T =25°C, ($\mu_m = 0.89\ mPa - s$)



Motivation

Trajectory captured by Data Logger, Quadrant Photodiode





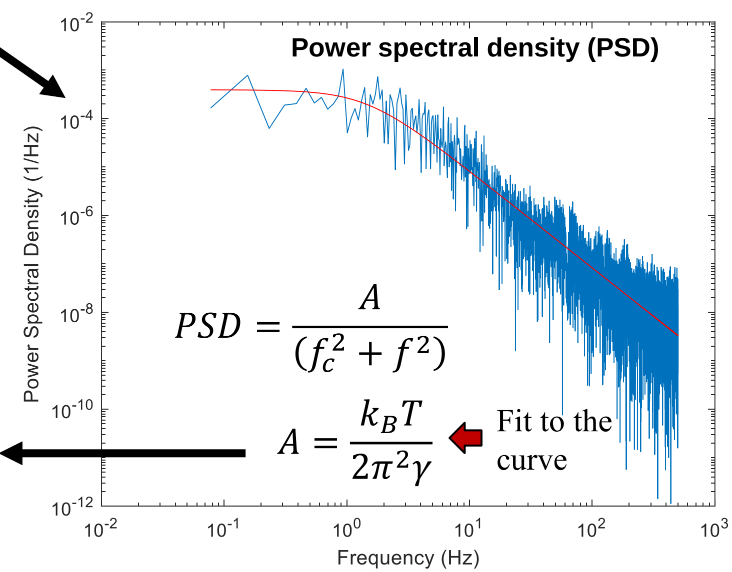
$$NACF = \exp(-t/\tau)$$

$$k_s = \frac{k_B T}{\langle r(0)\rangle^2}$$

$$\mu_{eff} = \frac{k_s \tau_c}{3\pi D}$$

$$f_c = {}^{1}/_{2\pi\tau}$$

$$\tau = \frac{\mu_0}{k_s}$$

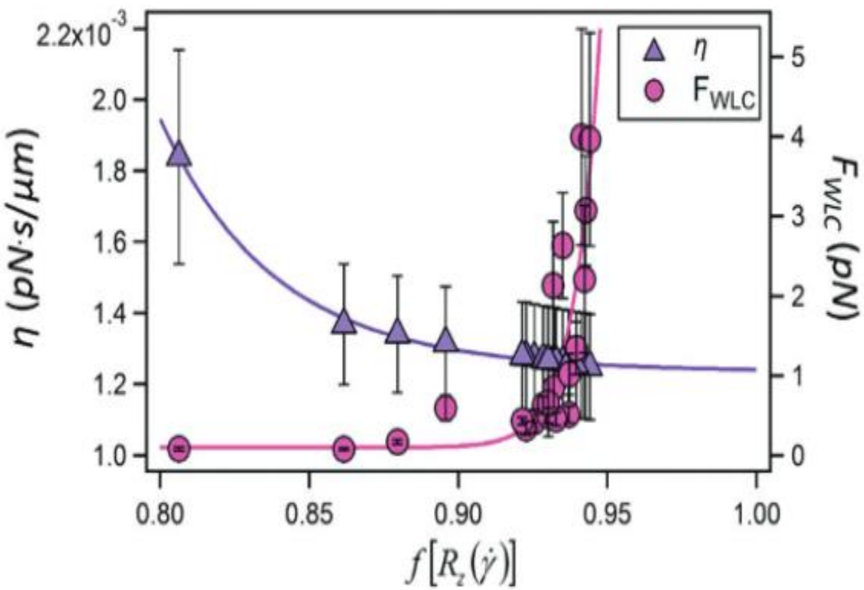


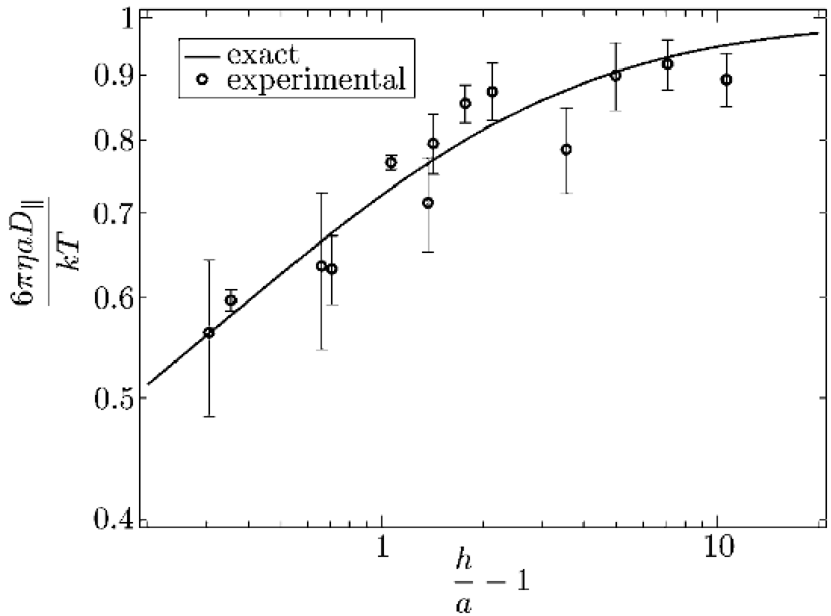
$$PSD = \frac{A}{(f_c^2 + f^2)}$$

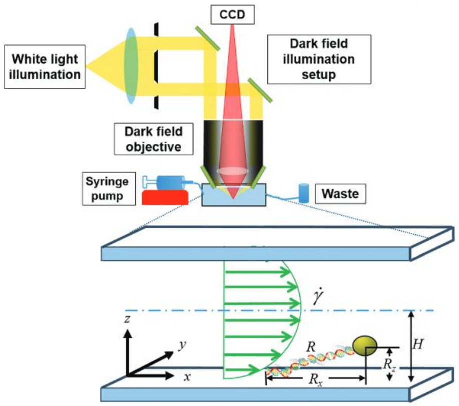
$$A = \frac{k_B T}{2\pi^2 \gamma}$$

$$k_s = 3\pi\mu_{eff} D/\tau \Leftarrow \gamma = 3\pi\mu_{eff} D$$

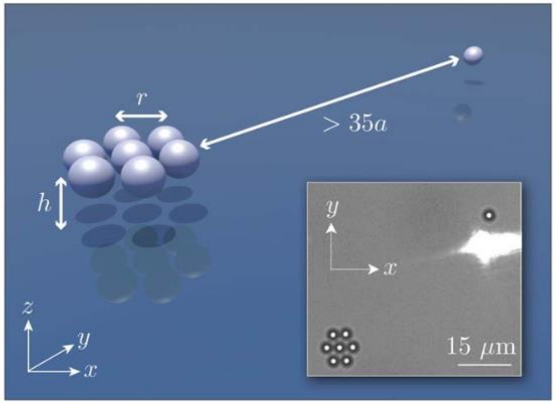
Motivation







Nir, Guy, Einat Chetrit et al. *Soft Matter* 14, 2018



Lele, Pushkar P., James W. Swan et al. *Soft Matter* 7, 2011.

Hydrodynamic interaction

- The effective viscosity measured in an optical trapping experiment is expressed as,

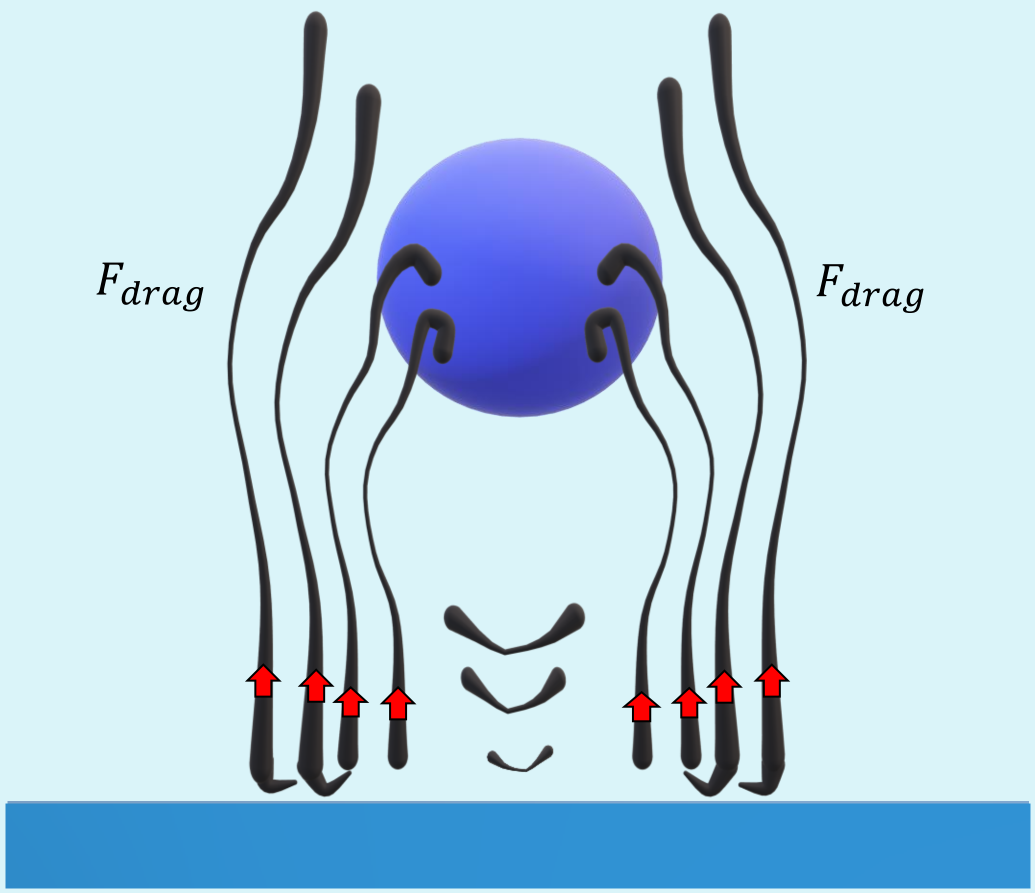
$$\mu_{eff} = \mu_0 \langle \boldsymbol{f} \rangle$$

- $f_{\parallel}(z)$ is a **correction** to the drag force due to the proximity of the wall (S.L. Eichmann, S.G. Anekal et al. Langmuir 24, 2008).

$$\mu_{eff} = \mu_0 f_{\parallel}(z)$$

- $f_{\perp}(z)$ is the perpendicular **correction** to the drag force due to the proximity of the wall M.A. Bevan, D.C. Prieve, Journal of Chemical Physics 113, 2000

$$\mu_{eff} = \mu_0 f_{|}(z)$$



Background Information

$$F_{grad,Z} = k_s \triangle z \frac{exp\left(\left(-\frac{z}{z_s}\right)^4\right)}{\left(1+\left(\frac{\triangle z}{z_s}\right)^2\right)^2}$$

Langevin equation governing the motion of the probe,

$$m\frac{dU}{dt}\cancelto{0}{} = -F_{drag} + F_C + F_B \qquad F_{drag} \gg m\frac{dU}{dt}$$

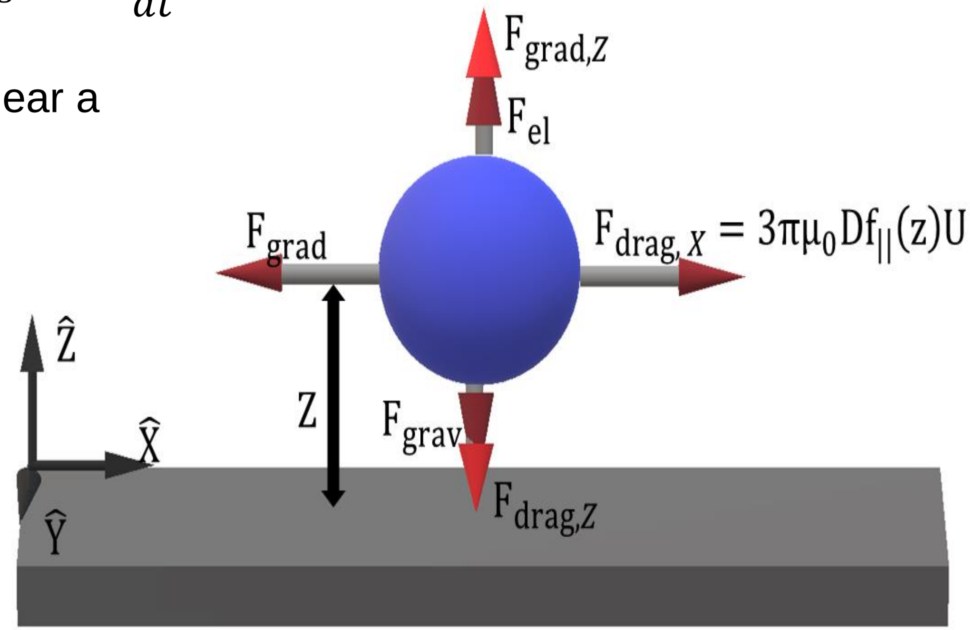
The drag force for a particle diffusing near a solid boundary in a creeping flow,

$$F_{drag} = 3\pi\mu_0 D\boldsymbol{f}\boldsymbol{U}$$

$$F_C = F_{grav} + F_{electrostatic} + F_{grad,Z}$$

$$F_{grav} = -\frac{4}{3}\pi a^3(\rho_p - \rho_f)g$$

$$F_{electrostatic} = \boldsymbol{\kappa} B_{pw}\exp(-\boldsymbol{\kappa}(z-a))$$



Background Information

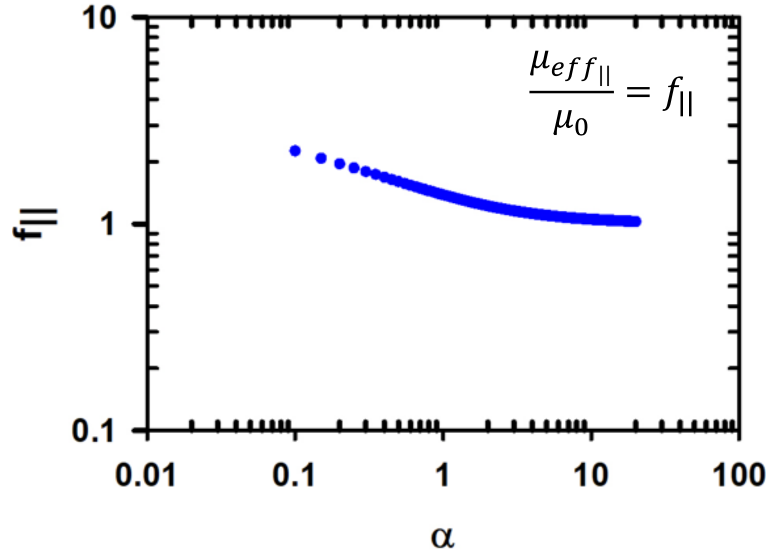
- Eichmann et al., Langmuir, 2007 fitted to an exact solution for $f_{||}(z)$ is,

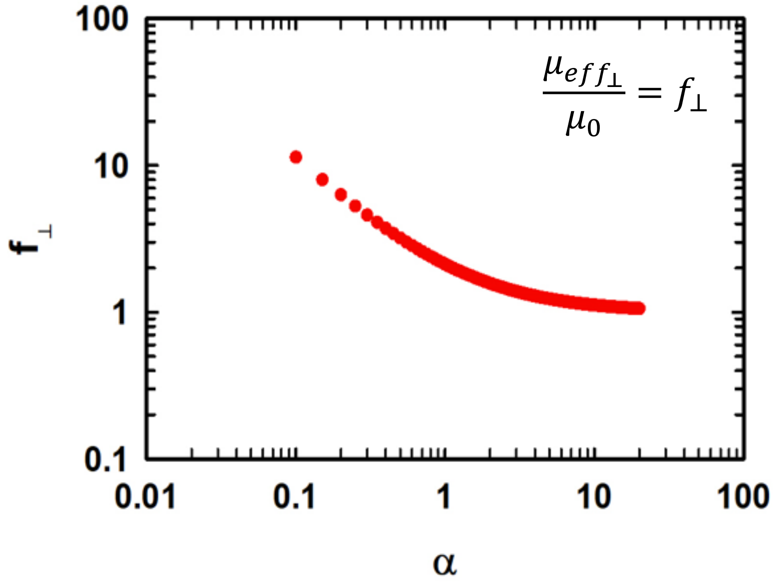
$$f_{||}(z) = \frac{12420\alpha^2 + 12233\alpha + 431}{12420\alpha^2 + 5654\alpha + 100}$$

M.A. Bevan, D.C. Prieve et al. Journal of Chemical Physics 113, 2000 fitted to an exact solution for $f_{\perp}(z)$, the perpendicular **correction** to the drag force due to the proximity of the wall

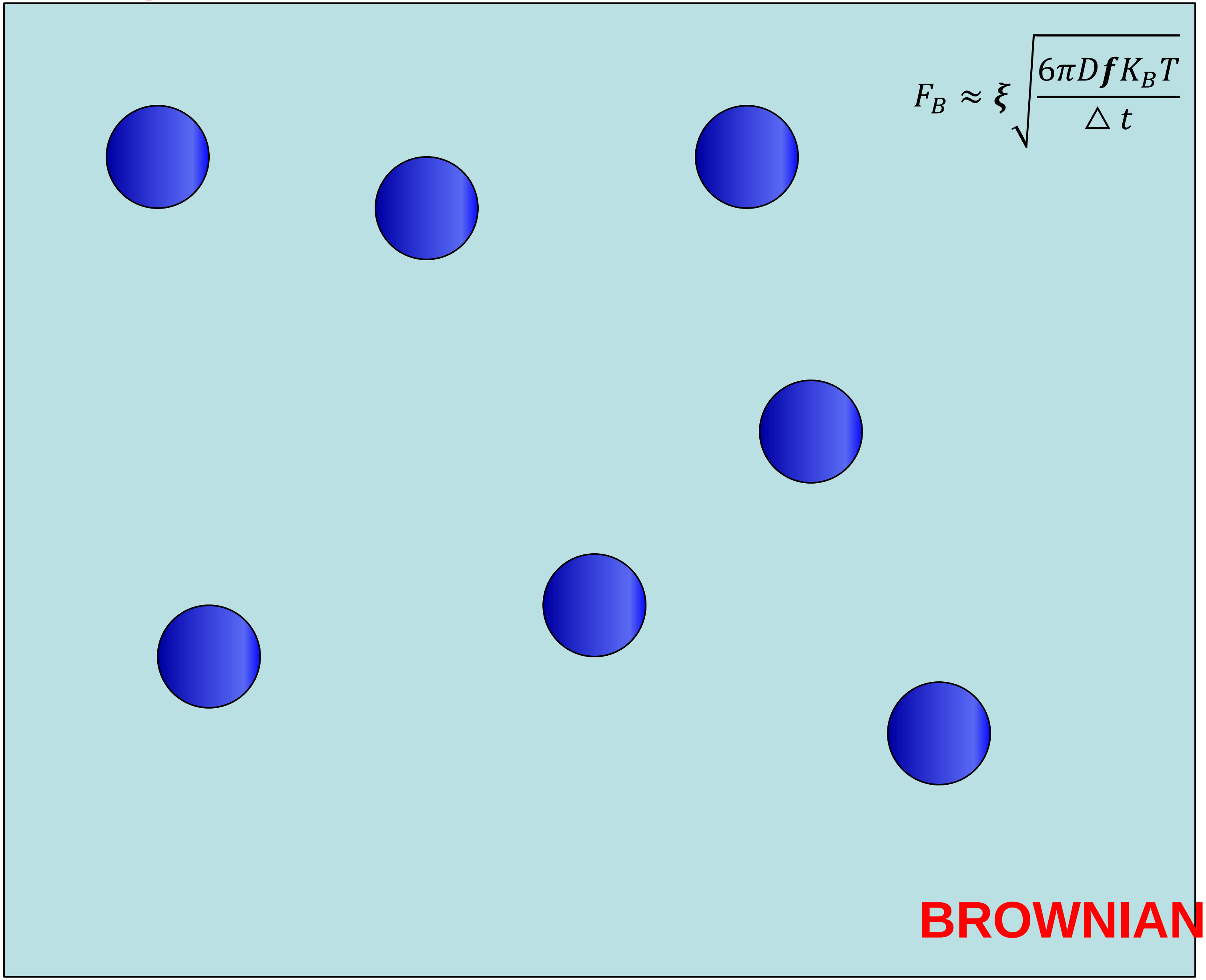
$$f_{\perp}(z) = \frac{6\alpha^2 + 9\alpha + 2}{6\alpha^2 + 2\alpha}$$

Where $\alpha = \frac{2z}{D} - 1$





Background Information



$$F_B \approx \boldsymbol{\xi}\sqrt{\frac{6\pi D \boldsymbol{f} K_B T}{\triangle t}}$$

BROWNIAN MOTION

Background Information

- Fixman, Marshall, *The Journal of Chemical Physics* 69,1978 utilized **midpoint algorithm** to numerically integrate the Langevin equation.

$$\boldsymbol{X}^{s} = \boldsymbol{X}^{n} + \frac{\Delta t}{2\lambda_0 \boldsymbol{f}^{n}}[F_c^n + F_B^n]$$

1. **Positional fractional step**

2. $\boldsymbol{f}^{s} = \boldsymbol{f}(\boldsymbol{X}^{s})$

Updated friction factors

3. $\boldsymbol{F}_c^{s} = \boldsymbol{F}_c(\boldsymbol{X}^{s})$

Updated conservative forces

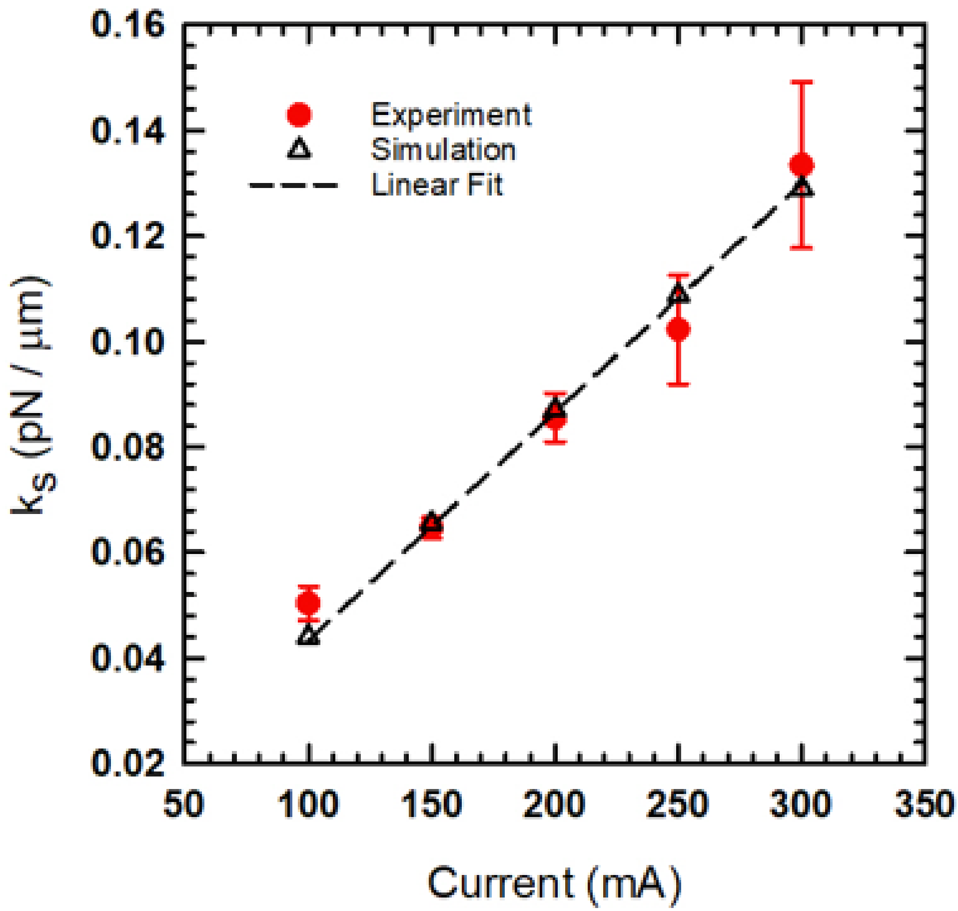
$$\boldsymbol{X}^{n+1} = \boldsymbol{X}^{n} + \frac{\Delta t}{\lambda_0 \boldsymbol{f}^{s}}[F_c^s + F_B^n]$$

F_B^n *not updated*

Results

- Data points for all graph shows average data for 10 independent experiments conducted on different beads.
- Data points for all graph shows average data for 100 independent simulation conducted.

$$k_s = \frac{k_B T}{\langle r(0) \rangle^2}$$



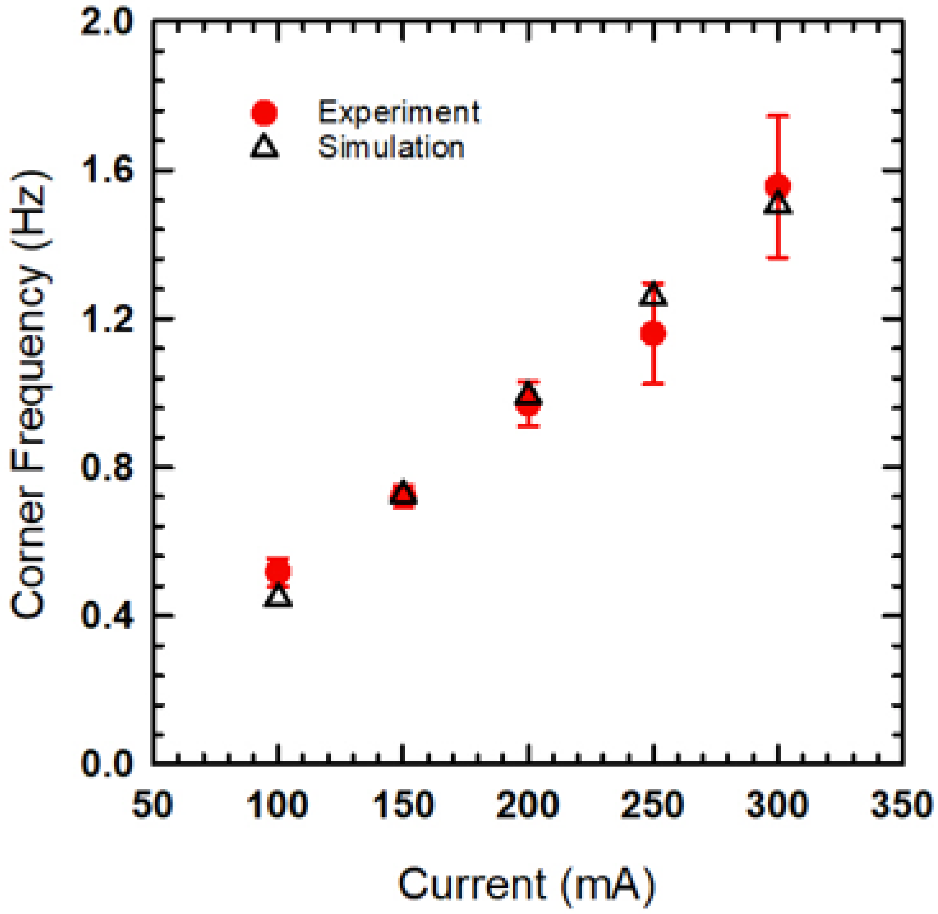
Results

Normalized Auto-correlation function

$$\frac{\langle r(0)r(t)\rangle}{\langle r(0)\rangle^2} = exp(-\tau/\tau_c)$$

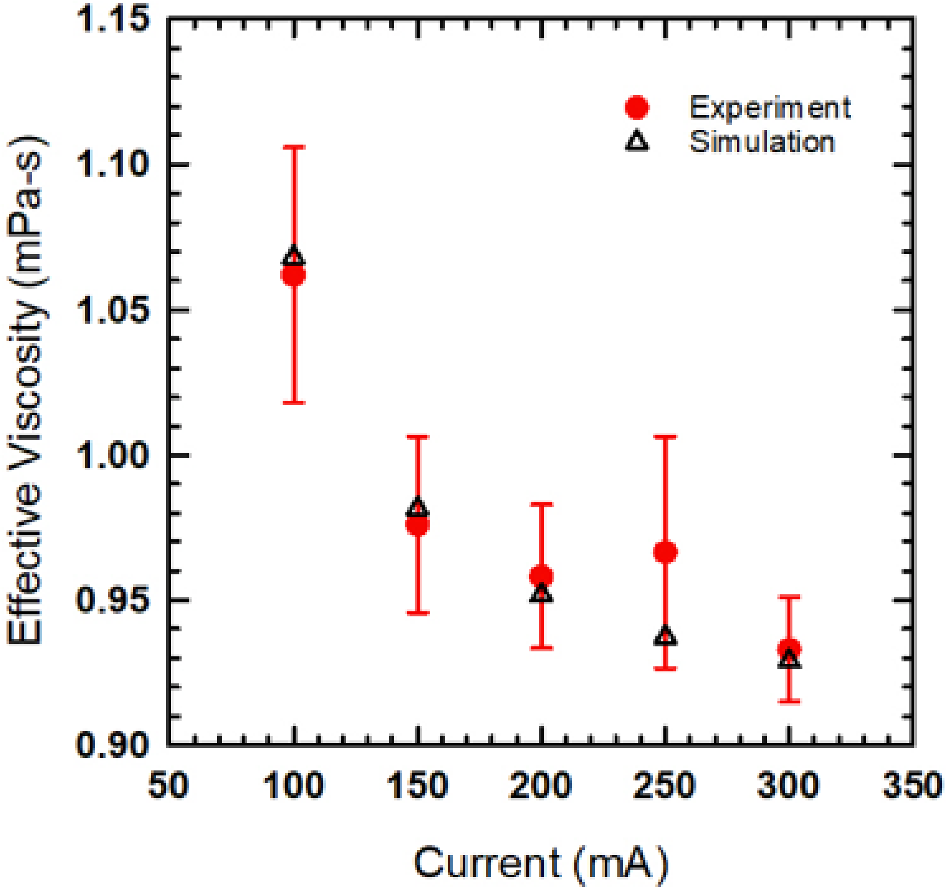
$$f_c = k_s/6\pi^2\mu_o D\langle \boldsymbol{f}\rangle$$

μ_0= solvent viscosity



Results

$$P(f) = \frac{k_B T}{2\gamma\pi^2 (f^2 + f_c^2)}$$



Summary

- A novel simulation technique has been developed considering the friction factors that emerges because of the hydrodynamic effect of the wall near to the focus of the laser.

- The results obtained from the simulation remains in close vicinity with that obtained from the experimental results.

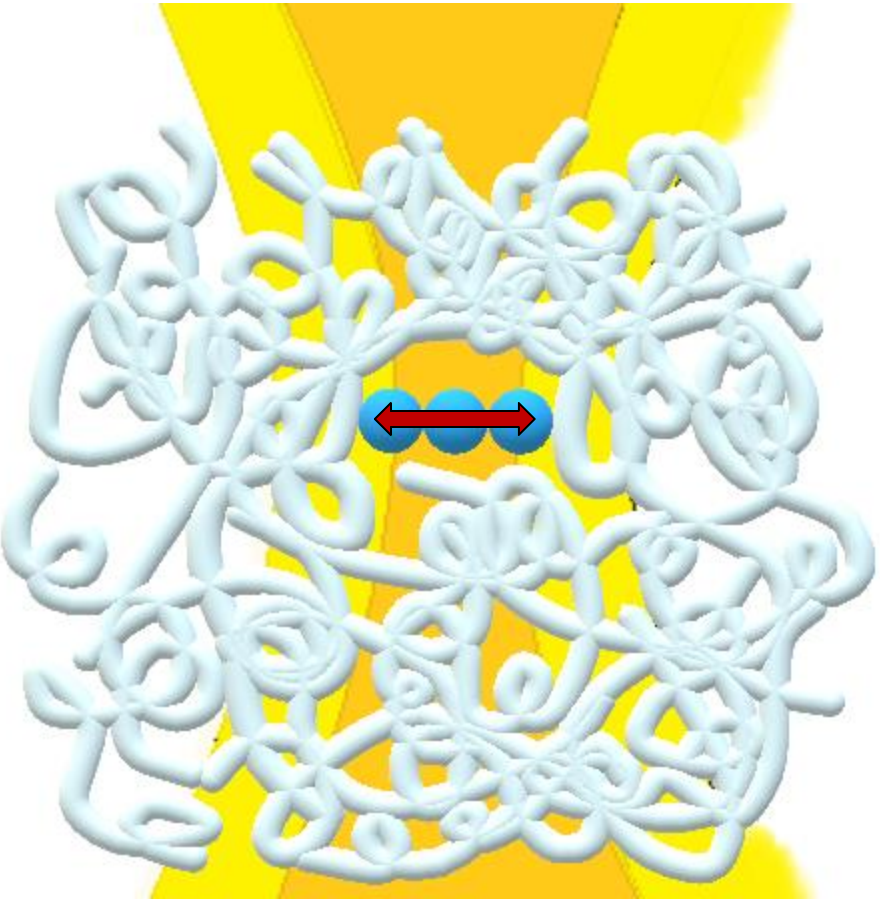
  1. Maintains the nature of the corner frequency as expected with the increase in laser power

  2. Linear increase in spring constant with that of laser power.

  3. Viscosity of the water being slightly in the higher range due to the wall effect.

Future work

- Hydrodynamic effect on optical trapping measurements for non-Newtonian fluids.
- Determining how the hydrodynamic effect changes with the variation of spacer thickness in between the probe particle and the surface of the cover glass.



Acknowledgement

We are grateful to National Science Foundation (NSF) Mechanics of Materials through award #2114565 for providing funding to attend the conference.

Questions ??

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